André Dabrowski's work on limit theorems and weak dependence

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Abstract: André Robert Dabrowski, Professor of Mathematics and Dean of the Faculty of Sciences at the University of Ottawa, died October 7, 2006, after a short battle with cancer. The author of the present paper, a long-term friend and collaborator of André Dabrowski, gives a survey of André's work on weak dependence and limit theorems in probability theory.

1. INTRODUCTION

Though almost 30 years have passed since then, I still vividly remember the scene where I met André Dabrowski for the first time. It was an early Saturday morning in the fall of 1979. We were standing in a crowd in front of the Illini Union building at the University of Illinois in Urbana, waiting for departure to the first Midwest Probability Colloquium, to be held that weekend at Northwestern University. André had already been a graduate student in Urbana for a year, I had just the week before arrived from Germany and did not know a single person in the crowd. It was here that André came up to introduce himself in his typical friendly and welcoming manner. The last time I met André in person was almost 27 years later, Saturday, July 1, 2006, in Graz, Austria. We both attended a small meeting organized in honor of Walter Philipp's 70th birthday. The scientific part of the meeting had ended the day before and this Saturday we held a small excursion to the Austrian Alps in the vicinity of Graz. Walter Philipp was with us, so were István Berkes and Robert Tichy. We were in a cheerful mood, among other things celebrating the first day of André's office as Dean of the Faculty of Sciences at the University of Ottawa. Less than three weeks later, we received news that Walter Philipp had died of a sudden heart attack. Another two weeks later, André told me that he had been diagnosed with cancer.

André Dabrowski and I have collaborated for over 25 years. Our collaboration began while both of us were graduate students of Walter Philipp at the University of Illinois. Still in our student days, we ran a small seminar discussing recent papers on weak dependence, Banach space valued random variables and functional limit theorems. In 1980 we began to work on our first joint paper, together with Walter Philipp, on the topic of almost sure invariance principles for triangular arrays of Banach space valued random variables. Since then, André and I always had a joint project going. We worked together while meeting at conferences and during numerous visits to each others home institutions. André visited me first in Göttingen, then in Boston, later in Groningen and finally in Bochum. I enjoyed frequent visits to Ottawa. All of our projects eventually resulted in a publications; in total we have published eight joint papers.

Born 1955 in London, U.K., André Robert Dabrowski grew up in Ottawa, where he attended school and in 1973 entered the University of Ottawa. André graduated with a B. Sc. and an M. Sc. in Mathematics in 1977 and 1978, respectively. André then obtained an NSERC graduate student fellowship that allowed him to go to graduate school in the US. Upon recommendation of his Masters' thesis advisor, Chandrakant Deo, André decided to go to the University of Illinois at Urbana-Champaign in order to work under the guidance of Walter Philipp. At that time, the probability and statistics group at the University of Illinois had a most impressive list of faculty members. Among the professors were, in addition to Walter Philipp, Don Burkholder, Catherine Doleans-Dade, Joe Doob, Frank Knight, Steve Portnoy, Bill Stout and Jack Wolfowitz. Walter Philipp was at that time internationally recognized as a leader in the development of limit theory for weakly dependent processes. At the University of Illinois, André first obtained an M. Sc. in Statistics in 1980 and then in 1982 his Ph. D. in Mathematics. Actually, André spent the second half of his time as a Ph.D. student at MIT in Cambridge, where Walter Philipp was on sabbatical from 1980 to 1982. From 1982 to 1985 André was Assistant Professor at the University of Calgary, Alberta. In 1985, André was offered a chance to return to his alma mater, the University of Ottawa, which he gladly accepted. In 1990 André was promoted to Associate Professor and finally in 1999 to Full Professor of Mathematics. In 2006, André was elected Dean of the Faculty of Sciences at the University of Ottawa. On October 7, 2006, André died in Ottawa after a short battle with cancer. He is survived by his wife, Deborah, and their two children, Adam and Leah.

In his scientific research, André Dabrowski combined deep theoretical work with a profound interest in applications. In the choice of his research topics, André was extremely adventurous. He was not afraid of having to learn new subjects and new techniques and especially valued interdisciplinary research very highly. On the theoretical side, André worked mostly on various types of limit theorems for dependent data. The range of topics was extremely broad, from partial sums, extremes, point processes and empirical processes for weakly dependent data to multiparameter martingales. On the applied side, André has worked on ion-channel modeling, statistical techniques for monitoring fetal heart rates, modeling of environmental data as spatial processes and DNA-microarray data analysis, to name just some topics. André took each of these applications very serious, really studying in depth the subject matter behind the application. At the same time, while keeping a high research profile, André was a dedicated academic teacher and an efficient and caring administrator. It was not by chance that the Faculty of Sciences of the University of Ottawa elected André as Dean!

In the next sections of this paper I will highlight some of André Dabrowski's major contributions in the areas of limit theorems and weak dependence. This was the area in which André worked as a graduate student and to which he returned throughout his career. This was also the area of our collaboration. What changed over the years was the motivation, from mostly theoretical interest in the beginning to needs arising in concrete applications in later years.

2. EARLY WORK ON LIMIT THEOREMS FOR PARTIAL SUMS

In order to understand the background of André Dabrowski's early work we have to recall the state of the art in limit theorems in probability and mathematical statistics in the late 1970's. Limit theory for independent real-valued random variables was quite developed by that time. Suppose $(X_i)_{i\geq 1}$ are i.i.d. random variables with mean zero and finite variance. Define the partial sum process $S_n : [0,1] \to \mathbb{R}$ by

$$S_n(t) := \begin{cases} \sum_{i=1}^k X_i & \text{if } t = \frac{k}{n} \\ \text{linearly interpolated} & \text{in between.} \end{cases}$$

Already in 1951, Donsker had established the functional central limit theorem, stating that the sequence of normalized partial sum processes $(\frac{S_n(\cdot)}{\sqrt{n}})_{n\geq 1}$, viewed as $C[0,1]$-valued random elements, converges in distribution to Brownian motion.

In 1964, Strassen had been able to formulate and prove a functional version of the law of the iterated logarithm. Strassen's theorem states that the sequence $(\frac{S_n(\cdot)}{\sqrt{2n \log\log n}})_{n\geq 1}$ is almost surely relatively compact and has as its set of limit points

$$K = \left\{ x \in C[0,1] : x \text{ absolutely continuous and } \int_0^1 (\dot{x}(t))^2 dt \leq 1 \right\}.$$

In the course of the proof of his functional law of the iterated logarithm, Strassen established an almost sure invariance principle. Strassen could show that one can redefine the process $(X_i)_{i \geq 1}$ on a possibly larger probability space together with Brownian motion $(W_t)_{t \in [0,\infty)}$ such that

$$\sum_{i=1}^{n} X_i - W_n = o(\sqrt{n \log \log n}).$$

With the help of this almost sure invariance principle, the functional law of the iterated logarithm for partial sum processes can be derived from the same result for Brownian motion, which was also established by Strassen (1964).

Though Strassen's almost sure invariance principle is helpful when establishing functional laws of the iterated logarithm, the error term is too big for a proof of Donsker's invariance principle. Thus naturally the question arose whether the $o(\sqrt{n \log \log n})$ error term could be improved. This question was completely settled in the mid 1970s in a series of papers written by authors from the Hungarian school of probabilists. Using the so-called quantile transform technique developed by M. Csörgő and Revesz (1975), Komlos, Major and Tusnady (1975, 1976) could show that an $o(n^{1/p})$ error rate is possible, provided the random variables X_i have finite p-th moment, $p > 2$. This is at the same time the optimal rate for $p > 2$, i.e. one can redefine the process $(X_i)_{i \geq 1}$ on a possibly larger probability space together with Brownian motion $(W_t)_{t \in [0,\infty)}$ in such a way that

$$\sum_{i=1}^{n} X_i - W_n = o(n^{1/p})$$

if and only if $E|X_i|^p < \infty$. In addition, Major (1976) could show that without extra moment conditions other than $EX_i^2 < \infty$, Strassen's original $o(\sqrt{n \log \log n})$ is optimal. This final result is disappointing, because it shows that Donsker's invariance principle cannot be obtained from an almost sure invariance principle unless one requires higher than second moments. In this situation, Major (1976) could show that an $o(n^{1/2})$ error term is possible, but only in probability. More precisely, Major (1976) showed that one can redefine the process $(X_i)_{i \geq 1}$ on a possibly larger probability space together with Brownian motion $(W_t)_{t \in [0,\infty)}$ in such a way that

$$\max_{1 \leq k \leq n} \left| \sum_{i=1}^{k} X_i - W_k \right| = o_P(n^{1/2}).$$

This invariance principle in probability immediately implies Donsker's invariance principle.

By the late 1970s, the above results were already history and pretty much settled the questions for i.i.d. real-valued observations with finite variance; see also the monograph by Csörgő and Révész (1981). New questions arose in the mid to late 1970s in connection with the rapid development in the following areas

- Probability in Banach spaces
- Weakly dependent processes
- Infinite variance processes, stable limit processes.

An obvious question was whether the classical limit theorems, obtained in the case of i.i.d. real-valued random variables with finite variance could be generalized to any of the new areas of research. The answer to this question was not at all obvious, since the classical proofs certainly could not be extended. E.g. the very powerful quantile transform technique developed by the Hungarian school was limited to the classical setup. In this situation, Walter Philipp and Istvan Berkes (1979) developed a new approximation technique that allowed to treat both dependent as well as Banach space valued random variables.

Before continuing, we have to introduce the basic concepts of the theory of weakly dependent processes. Let $(\Omega, \mathcal{F}, P)$ be a probability space. For sub-σ-fields $\mathcal{A}, \mathcal{B} \subset \mathcal{F}$, we define the following measures of the degree of dependence of $\mathcal{A}$ and $\mathcal{B}$,

$$\begin{aligned}
\beta(\mathcal{A},\mathcal{B}) &:= E(\sup_{B\in\mathcal{B}} |P(B|\mathcal{A}) - P(B)|) \\
\phi(\mathcal{A},\mathcal{B}) &:= \sup_{A\in\mathcal{A},B\in\mathcal{B}} |P(B|A) - P(B)| \\
\psi(\mathcal{A},\mathcal{B}) &:= \sup_{A\in\mathcal{A},B\in\mathcal{B}} \frac{|P(A\cap B) - P(A)\,P(B)|}{P(A)\,P(B)}
\end{aligned}$$

Given a process $(X_i)_{i\geq 1}$, define the σ-fields $\mathcal{F}_k^l := \sigma(X_k, \ldots, X_l)$ and the mixing coefficients

$$\beta(k) := \sup_n \beta(\mathcal{F}_1^n, \mathcal{F}_{n+k}^\infty),$$

and similarly $\phi(k)$ and $\psi(k)$.

Defintion 1. The process $(X_i)_{i\geq 1}$ is called absolutely regular (or β-mixing), if $\lim_{k\to\infty} \beta(k) = 0$. Similarly, the process is called uniformly mixing (or ϕ-mixing) if $\lim_{k\to\infty} \phi(k) = 0$ and ψ-mixing if $\lim_{k\to\infty} \psi(k) = 0$.

The Berkes-Philipp technique provides an approximation of a sequence $(X_k)_{k\geq 1}$ of dependent random variables with values in a separable metric space by independent random variables $(Y_k)_{k\geq 1}$ with possibly different, but close, marginal distributions. The quality of approximation depends on the degree of dependence of the $(X_k)_{k\geq 1}$-process as well as on the Prohorov distance of the marginal distributions.

Theorem 1. (Berkes & Philipp 1979) *Let, for each $k \geq 1$, (S_k, σ_k) be a complete separable metric space, X_k an S_k-valued random variable with probability distribution F_k and let G_k be another probability distribution on S_k. Define moreover*

$$\phi_k := \phi(\mathcal{F}_1^k, \mathcal{F}_{k+1}^\infty).$$

Then, after possibly enlarging the probability space, we can redefine the sequence $(X_k)_{k\geq 1}$ without changing its joint distribution together with a sequence of independent rv's $(Y_k)_{k\geq 1}$ such that Y_k has distribution G_k and

$$P(\sigma_k(X_k, Y_k) \geq 6\phi_k + \pi(F_k, G_k)) \leq 6\phi_k + \pi(F_k, G_k).$$

Here $\pi(F_k, G_k)$ denotes the Prohorov distance of F_k and G_k.

The Berkes-Philipp approximation theorem provided a very powerful technique for attacking problems in various areas, e.g. vector-valued processes and empirical processes of dependent data. This is the background against which André Dabrowski in 1978 started his Ph.D. research under the guidance of Walter Philipp. Actually, André's very first paper was submitted even before that, and it contained results from his Master's thesis, published jointly with his thesis advisor, Chandrakant Deo. The topic of the paper was r-quick convergence in Strassen's functional law of the iterated logarithm for dependent processes. Roughly speaking, r-quick convergence says that the first time the process $(\frac{S_n(\cdot)}{\sqrt{2n\log\log n}})_{n\geq 1}$ comes close to a given function in the Strassen class has finite r-th moment. Dabrowski & Deo (1981) could show this under certain weak dependence conditions.

The first paper that André wrote during his Ph.D. studies was entitled *A note on a theorem of Berkes and Philipp for dependent sequences.* In this paper André could improve an invariance principle for ϕ-mixing processes estables by Berkes and Philipp (1979) as an application of their approximation technique.

THEOREM 2. (Dabrowski 1982) *Let $(X_i)_{i\geq 1}$ be a ϕ-mixing strictly stationary process, $E(X_i)=0$, $E|X_i|^{2+\delta}<\infty$, for some $\delta>0$, satisfying*

$$\phi(n) = O((\log n)^{-(1+\epsilon)(1+2/\delta)})$$

and $Var(\sum_{i=1}^n X_i)\to\infty$. Then, after possibly enlarging the probability space, one can find standard Brownian motion $(W_t)_{t\in[0,\infty)}$ such that

$$\sum_{i=1}^n X_i - \sigma W_n = O(\sqrt{n/\log\log n}) \ \ a.s.$$

where $\sigma^2 = \lim_{n\to\infty}\frac{1}{n}Var(\sum_{i=1}^n X_i)$.

The first two papers of André Dabrowski that I want to discuss in some detail, are joint papers with Walter Philipp, me and in one case also Istvan Berkes. Both papers treat i.i.d. processes of vector-valued random variables with possibly infinite variances. One of the papers investigates an almost sure invariance principle for triangular arrays of random variables converging to some infinitely divisible law, the other one an almost sure invariance principle for partial sums in the domain of attraction of a stable law.

THEOREM 3. (Dabrowski, Dehling & Philipp 1984) *Let $(\mu_n)_{n\geq 1}$ be a sequence of probability measures on the separable Banach space B, and let k_n be a sequence of integers such that $k_n\to\infty$ and*

$$\mu_n^{*k_n}\to\mu,$$

where $*$ denoted convolution power. Then there exists a probability space $(\Omega,\mathcal{F},P)$ and two row-wise independent triangular arrays of random variables $(X_{nk})_{1\leq k\leq k_n, n\geq 1}$ and $(Y_{nk})_{1\leq k\leq k_n, n\geq 1}$, defined on $(\Omega,\mathcal{F},P)$ such that

$$\mathcal{L}(X_{nk}) = \mu_n, \quad \mathcal{L}(Y_{nk}) = \mu^{*\frac{1}{k_n}},\ 1\leq k\leq k_n,$$

and

$$\max_{k\leq k_n}\left\|\sum_{j=1}^k (X_{nj}-Y_{nj})\right\|\to 0 \ \ a.s.$$

A weaker version of this theorem, with convergence in probability instead of almost sure convergence, had been established before by De Acosta (1982). Our goal was, among other things, to show that the Berkes-Philipp technique would provide an alternative and more direct proof of De Acosta's theorem.

Theorem 4. (Berkes, Dabrowski, Dehling & Philipp 1986) *Let $(X_j)_{j\geq 1}$ be i.i.d. $\mathbb{R}^d$-valued random variables, in the domain of normal attraction of the stable law G_α, $0<\alpha<2$, satisfying*

$$\int_{\mathbb{R}^d} \|x\|^{1+[\alpha]}|F-G_\alpha|(dx)<\infty.$$

Then, after possibly enlarging the probability space, one can define an i.i.d. process $(Y_i)_{i\geq 1}$ of G_α-distributed random variables such that

$$\sum_{i=1}^{n}(X_i-Y_i)=O(n^{1/\alpha-\lambda}),$$

where $\lambda=\frac{1-\{\alpha\}}{32(d+1)\alpha}$.

Though these two theorems address two completely different problems, the important ideas in the proofs are rather similar. Both papers were only possible because of an ingenious idea that André contributed at a crucial stage. In order to explain this idea, we have to sketch the usual procedure for proving any kind of almost sure invariance principle. We consider the situation treated in Theorem 4. Given the process $(X_i)_{i\geq 1}$, we introduce blocks of consecutive integers $H_1, H_2, \ldots$ by

$$H_k=(t_{k-1},t_k],$$

where $0=t_0<t_1<t_2<\ldots$ and define $N_k=t_k-t_{k-1}=\#H_k$. The blocks increase in size as $k\to\infty$; whether polynomially, subexponentially or exponentially, depends on the given context. We then define the normalized block sums

$$V_k:=\frac{1}{N_k^{1/\alpha}}\sum_{i\in H_k}X_i.$$

Next, we apply bounds on the speed of convergence of V_k to the limit distribution G_α as measured with respect to the Prohorov distance

$$\rho_k=\pi(\mathcal{L}(V_k),\mu).$$

In the case of Theorem 4, we could apply results due to Banys (1976). By the Berkes-Philipp theorem, we can then find G_α-distributed random variables W_k satisfying

$$P(\|V_k-W_k\|\geq\rho_k)\leq\rho_k,$$

and such that W_k is independent of $W_1,\ldots,W_{k-1}$. In the next step, we define an i.i.d. process $(Y_i)_{i\geq 1}$ of G_α-distributed random variables such that

$$W_k=N_k^{-1/\alpha}\sum_{i\in H_k}Y_i.$$

This is possible, provided the underlying probability space is large enough, because the joint distributions of the normalized block sums $(N_k^{-1/\alpha}\sum_{i\in H_k}Y_i)_{k\geq 1}$ equal equal those of $(W_k)_{k\geq 1}$. The technical details are provided by a result of Skorohod (1976).

By construction of the processes, one has control over the distance of the partial sums at the ends of the blocks, because

$$\sum_{i=1}^{t_k}(X_i-Y_i)=\sum_{j=1}^{k}N_j^{1/\alpha}(V_j-W_j).$$

In order to bound the distances at every time point, the usual approach is to use the triangle inequality,

$$\max_{n\in H_k}\left\|\sum_{i=t_{k-1}+1}^{n}(X_i-Y_i)\right\| \le \max_{n\in H_k}\sum_{i=t_{k-1}+1}^{n}\|X_i\| + \max_{n\in H_k}\sum_{i=t_{k-1}+1}^{n}\|Y_i\|$$

In all previous proofs of almost sure invariance principles, one could then control the right hand side with the help of a suitable maximal inequality. We tried very hard to make this work also in the situation of Theorem 3 and Theorem 4, but the calculations never worked out. It was finally André who realized that the reason for this was much deeper. It was not our inability to get the calculations straight, but there was an inherent problem related to the fact that the limit process in infinite variance limit theorems necessarily has jumps. At the same time, André found the solution: construct the random variables $Y_i, i \in H_k$ in such a way that the location of its largest absolute value, and therefore the location of the largest jump in the partial sum process, matches that of the $X_i, i \in H_k$. It is not at all obvious that this could work, but it does using some subtle arguments. Roughly speaking this can be done, because the location of the largest absolute value is independent of the partial sum.

In two later papers, André Dabrowski was able to extend the last two theorems to the case of weakly dependent observations. In his 1987 paper in the Canadian Journal of Statistics, André could establish an invariance principle in probability for Banach-space valued ϕ-mixing processes in the domain of attraction of a stable law. In Dabrowski & Zoglat (1995) the above Theorem 3 is extended to a class of weakly dependent observations.

3. LIMIT THEOREMS FOR ASSOCIATED SEQUENCES

In the early 1980s André Dabrowski began to investigate limit theorems for associated processes. Association is a notion of weak dependence that had been introduced in the 1960s independently in reliability theory as well as in interacting particle systems.

Definition 2. (Esary, Proschan & Walkup 1967) A sequence $(X_i)_{i\ge 1}$ of real-valued random variables is called *associated* if

$$\mathrm{Cov}(f(X_1,\ldots,X_n), g(X_1,\ldots,X_n)) \ge 0$$

for all coordinate-wise increasing functions $f, g : \mathbb{R}^n \to \mathbb{R}$ satisfying $E(f(X_1,\ldots,X_n))^2 < \infty$ and $E(g(X_1,\ldots,X_n))^2 < \infty$ and for all ≥ 1.

By itself, association is not necessarily related to weak dependence. However, association provides a link between the decay of correlations and the degree of dependence, much in the same way as for Gaussian processes. The link was discovered in 1980 by Charles Newman, the early pioneer in the field of limit theorems for associated processes.

Lemma 1. (Newman 1980) *Let $Y_1,\ldots,Y_n$ be associated random variables with finite variances. Then*

$$|\mathrm{E}\exp(i\sum_{k=1}^{n}t_kY_k) - \prod_{k=1}^{n}\mathrm{E}\exp(it_kY_k)| \le \sum_{1\le j<k\le n}|t_jt_k|\mathrm{Cov}(Y_j,Y_k).$$

for all $t_1,\ldots,t_n \in \mathbb{R}$.

Thus the difference between the joint characteristic function of the random variables and the product of the characteristic functions, which would be the characteristic function if the Y_k were independent, can be controlled by the covariances of the process. Typically, this inequality is applied to block sums of the original X_j-random variables.

With the help of this lemma, Newman (1980) could prove a first central limit theorem for associated processes, provided the covariances are summable. In this case, the limit variance

$$\sigma^2 = \mathrm{Var}(X_1) + 2\sum_{k=1}^{\infty} \mathrm{Cov}(X_1, X_{k+1})$$

exists and is finite. Under the same conditions, Newman & Wright (1981) established the functional central limit theorem. Finally, Wood (1983) proved a Berry-Esseen inequality, assuming higher moments.

Given these results, it was a natural goal to establish the law of the iterated logarithm for associated processes. In the light of the work of Walter Philipp and his group, a natural approach was to establish an almost sure invariance principle for associated processes, i.e. to show that one could find an i.i.d. process Z_i of normally distributed random variables satisfying

$$\sum_{i=1}^{n}(X_i - Z_i) = o(\sqrt{n \log\log n}).$$

I know that André tried this, we also discussed this quite intensely, e.g. at the 1985 Oberwolfach conference on Dependence in Probability and Statistics, but there were technical difficulties that could not be overcome at that time. It was only in 1996 that Hao Yu could establish an almost sure invariance principle for associated sequences.

André instead applied a technique that had been developed by István Berkes (1973) in the context of random trigonometric series. Effectively, this technique used joint characteristic functions and an exponential inequality. In this way, André could prove the following theorem:

Theorem 5. (Dabrowski 1985) *Let $(X_i)_{i\geq 1}$ be a non-degenerate strictly stationary associated sequence of mean-zero random variables satisfying*

$$\sup_{k\geq 1} \mathrm{E}\,|S_k/\sqrt{k}|^3 < \infty$$
$$\sigma^2 - \sigma_n^2 = O(n^{-\delta}).$$

Then $(X_i)_{i\geq 1}$ satisfies the functional law of the iterated logarithm, i.e.

$$\left(\frac{S_n(\cdot)}{\sqrt{2\,n\,\log\log n}}\right)_{n\geq 1}$$

is almost surely relatively compact with set of limit points

$$\{x \in C[0,1] : x(0) = 0 \text{ and } \int_0^1 (\dot{x}(t))^2 dt \leq \sigma^2\}.$$

The next step in the development of limit theory for associated processes was the extension to vector-valued random variables. First, a sensible definition of multidimensional associated processes had to be found. Of course, one could have literally copied the one-dimensional definition and this would have made perfect sense. However, it would have implied positive dependence of the coordinates within the same vector, e.g. of $X_{11}, \ldots, X_{1d}$ where $X_1 = (X_{11}, \ldots, X_{1d})$. This seemed too strong a requirement in many applications and thus a weaker condition had to be found.

Definition 3. (Burton, Dabrowski & Dehling 1986) A sequence $(X_i)_{i\geq 1}$ of $\mathbb{R}^d$-valued random vectors is called weakly associated if

$$\mathrm{Cov}(f(X_{\pi(1)},\ldots,X_{\pi(m)}),g(X_{\pi(m+1)},\ldots,X_{\pi(m+n)}))\geq 0,$$

for all positive integers n,m, all permutations π of the positive integers and all coordinate-wise non-decreasing functions $f:\mathbb{R}^{md}\to\mathbb{R}$, $g:\mathbb{R}^{nd}\to\mathbb{R}$ with finite variances.

This definition is essentially essentially due to André. We discussed it when André visited me at the University of Göttingen in the spring of 1985. Bob Burton was also in Göttingen at that time and together we were able to prove a multivariate CLT, and even a Donsker type invariance principle for the partial sum process.

Theorem 6. (Burton, Dabrowski & Dehling 1986) *Let $(X_i)_{i\geq 1}$ be a strictly stationary weakly associated process of $\mathbb{R}^d$-valued random vectors with $\mathrm{E}(X_1)=0$ and $\mathrm{E}\|X_1\|^2<\infty$ satisfying*

$$\sigma_{kl}:=\mathrm{Cov}(X_{1k},X_{1l})+\sum_{i=2}^{\infty}(\mathrm{Cov}(X_{1k},X_{il})+\mathrm{Cov}(X_{1l},X_{ik}))<\infty$$

Then the functional central limit theorem holds, i.e. the normalized partial sum process

$$\frac{1}{\sqrt{n}}(S_n(t))_{0\leq t\leq 1}\xrightarrow{\mathcal{D}}(W_t)_{0\leq t\leq 1},$$

where $(W_t)_{0\leq t\leq 1}$ is $\mathbb{R}^d$-valued Brownian motion satisfying $W_1\sim N(0,\Sigma)$.

The proof of this theorem required an interesting new technique. Naturally, we tried the Cramér-Wold device, which is the standard method to prove $\mathbb{R}^d$-valued limit theorems. However, linear combinations of the coordinates of the vectors are themselves not necessarily associated. This is only the case if all the weights are non-negative. We thus needed a generalization of the Cramér-Wold device with only non-negative weights. We could indeed show that such a non-standard Cramér-Wold device holds, provided the limit process has finite moment generating functions, which is the case for normal limits.

In a subsequent paper, written mostly in the period from 1985 to 1987, when André had just returned to Ottawa and when I was postdoc at Boston University, we could establish a functional law of the iterated logarithm for weakly associated processes, effectively combining the techniques of our CLT paper with André's FLIL paper for associated processes.

Theorem 7. (Dabrowski & Dehling 1988) *Let $(X_i)_{i\geq 1}$ be a strictly stationary weakly associated process of mean-zero $\mathbb{R}^d$-valued random variables, satisfying for some $\epsilon,\delta>0$*

$$E(\|X_i\|^{2+\epsilon})<\infty$$

$$\sigma_{kl}-\frac{1}{n}Cov(\sum_{i=1}^{n}X_{ik},\sum_{i=1}^{n}X_{il})=O(n^{-\delta}).$$

Then the functional law of the iterated logarithm holds, i.e.

$$\left(\frac{S_n(\cdot)}{\sqrt{2n\log\log n}}\right)$$

is almost surely relatively compact with set of limit points

$$K_A=\{x\cdot\Sigma^{1/2}:x\in C_d[0,1],\ abs.\ cont.,\int_0^1\|\dot{x}\|^2(t)dt\leq 1\}$$

André wrote two more papers on associated processes, one together with Bob Burton, the other with Adam Jakubowski. Bob Burton and André Dabrowski could prove a surprising result, namely that infinite exchangeable sequences of binary random variables are necessarily positively dependent, in the sense that they satify the strong FKG inequality.

The 1984 Annals of Probability paper, coauthored by André Dabrowski and Adam Jakubowski, investigates stable limits for associated processes. Since the usual work on associated processes uses covariances in a very strong way, it a priori not at all clear how to attack the infinite variance case. Though I did not participate in this project, I can pride myself of having been present while the work was initiated: André spent a sabbatical semester in the fall of 1990 in Groningen, where I had just arrived, and we invited Adam Jakubowski for a one week visit, during which they started this research.

Around that time, i.e. the early 1990s, a new generation of probabilists took over the torch in the area of limit theorems for associated processes. They could prove theorems we had been unable to prove. Most notably is Hao Yu's thesis work, written at Carleton University, under the guidance of Miklos Csörgő, on the empirical process invariance principle for associated sequences; see Hao Yu (1993). Recently, some very sharp results on this topic have been obtained by Sana Louhichi (2000), a young member of the Paris school of weak dependence.

4. POINT PROCESSES OF DEPENDENT DATA

André Dabrowski became interested in point process theory already in the early 1980s, while he was still a Ph. D. student and he continued working on this topic throughout his entire career. Point processes arise in a variety of contexts, e.g. in the study of extreme values. Suppose $(\xi_i)_{i\geq 1}$ is an i.i.d. process of real-valued random variables satisfying

$$P(\max(\xi_1, \ldots, \xi_n) - b_n \leq x\, a_n) \to G(x),$$

for some sequences $(a_n)_{n\geq 1}$ and $(b_n)_{n\geq 1}$. It is well known that in this case G belongs to one of three different types of extreme value distributions; see e.g. Resnick (1987). The modern theory of extreme values is closely related to the planar point process I_n obtained by placing unit mass into each of the points

$$(\frac{i}{n}, \frac{\xi_i - b_n}{a_n}),\ 1 \leq i \leq n.$$

Pickands (1971) showed that this point process converges weakly to a Poisson point process.

Extensions to certain weakly dependent processes have been obtained e.g. by Leadbetter (1974) and by Adler (1978). These theorems require a different type of weak dependence conditions than introduced before in connection with convergence of partial sums. Roughly speaking, these conditions, known as conditions (D) and (D'), specify that the occurence of joint extreme events $\xi_1 > x, \xi_k > x$ behaves asymptotically as if the variables were independent. In his first paper on point processes, André Dabrowski augmented the results of Adler and Leadbetter to an invariance principle.

Theorem 8. (Dabrowski 1990) *Let (ξ_i) be a stationary absolutely regular process with distribution function F satisfying, for all $x \in \mathbb{R}$,*

$$F^n(a_n\, x + b_n) \to G(x),$$

and condition (D'). Then there exists an i.i.d. process η_n of G-distributed random variables such that the associated point processes I_n and J_n satisfy

$$d(I_n, J_n) \xrightarrow{P} 0,$$

where d is a Skorohod-type metric for planar point processes.

The proof of Theorem 8 consists of a very clever application of the Berkes-Philipp approximation theorem, albeit in a sharper version suitable for absolutely regular processes due to Dehling and Philipp (1982). As an interesting application of Theorem 8, André could extend results of Csörgő and Horváth (1987) on intermediate quantile functions to weakly dependent processes.

For several years André took the occasion of spring break at the University of Ottawa in order to visit me in Europe and to get a chance to concentrate on research again. It was during one of these visits, in the spring of 2000, that, together with my Groningen colleague Thomas Mikosch and Olimjon Sharipov, a visitor from Tashkent, we began discussing the possibility of extreme value theory for U-statistics. Given an i.i.d. process $(X_i)_{i\geq 1}$ of random variables with values in some measurable space $\mathcal{X}$ and a kernel function $h : \mathcal{X}^2 \to \mathbb{R}$, we define the U-statistics

$$U_n(h) = \frac{1}{\binom{n}{2}} \sum_{1\leq i<j\leq n} h(X_i, X_j).$$

The asymptotic deistribution of $U_n(h)$ is well understood, even in the case of weakly dependent processes (X_i). Extreme values of the individual summands, however, had not been studied before. We took kernels h with nonnegative values and investigated lower extreme values of the sample

$$h(X_i, X_j),\ 1 \leq i < j \leq n.$$

Knowing the asymptotic distribution of these values is important in statistical inference for the lower tail of the distribution of $h(X_1, X_2)$.

Example. Consider the kernel $h(x, y) = \|x - y\|$, $x, y \in \mathbb{R}^d$. We say that the distribution of X_1 has correlation dimension α, if

$$P(\|X - Y\| \leq x) = x^\alpha L(x^{-1}),$$

for some $\alpha > 0$ and some slowly varying function $L : (0, \infty) \to (0, \infty)$. Correlation dimension estimation has been investigated by a number of authors, e.g. by Grassberger & Procaccia (1984). In practice, inference on α is based on the small values of $\|X_i - X_j\|$, $1 \leq i < j \leq n$ and thus we have to know their asymptotic distribution.

Motivated by the point process approach to classical extreme value theory, we considered the point process of U-statistics extreme values, defined as follows. Take the $\binom{n}{2}$ random points

$$\left(\frac{i}{n}, \frac{j}{n}, a_n\, h(X_i, X_j)\right),\ 1 \leq i < j \leq n,$$

where a_n will be defined below, and consider the associated random measure $N_n(\cdot)$, placing unit mass in each of these points. Then N_n is a random measure on the set

$$E = \{(x, y, z) : 0 < x < y \leq 1, z \geq 0\}.$$

Weak convergence of N_n requires two assumptions. The first is a condition on the lower tail of $h(X_1, X_2)$

(A1) For some slowly varying $L(\cdot)$ and $\alpha > 0$,

$$P(h(X_1, X_2) \leq x) = L(x^{-1})x^\alpha.$$

We then define the normalizing constants a_n such that

$$P(h(X_1, X_2) \le a_n^{-1}) \sim \frac{2}{n^2}.$$

The second assumption states that the occurence of joint extremes of $h(X_1, X_2)$ and $h(X_1, X_3)$ has small probability.

(A2) For any $x > 0$, as $n \to \infty$,

$$n^3 P(a_n h(X_1, X_2) \le x, a_n h(X_1, X_3) \le x) \to 0.$$

THEOREM 9. (Dabrowski, Dehling, Mikosch & Sharipov 2002) *If assumptions (A1) and (A2) hold,*

$$N_n \xrightarrow{D} N,$$

where N is a Poisson point process on E with intensity measure η given by

$$\eta((a_1, b_1] \times (a_2, b_2] \times (a_3, b_3]) = 2(b_1 - a_1)(b_2 - a_2)(b_3^\alpha - a_3^\alpha)$$

As an application of this theorem we could establish consistency of the Hill estimator for the tail index α of the distribution of $h(X_1, X_2)$. Define the order statistics of $h(X_i, X_j)$, $1 \le i < j \le n$,

$$h_{(1)} \le h_{(2)} \le \ldots \le h_{\binom{n}{2}}.$$

Then the Hill estimator of the tail index α is given by

$$\hat{\alpha}_{n,m} := -\left(\frac{1}{m} \sum_{i=1}^{m} \log(h_{(i)}/h_{(m)}) \right)^{-1}.$$

By representing the Hill estimator as a continuous function of the point process N_n, we could show that $\hat{\alpha}_{n,m} \xrightarrow{P} \alpha$, provided $m = m_n \to \infty$, $\frac{\sqrt{m_n}}{n} \to 0$ and assumptions (A1) and (A2) hold.

A very interesting open question concerns the extension of the above results to the case of dependent processes $(X_i)_{i \ge 1}$. This is highly relevant in applications, e.g. in time series analysis.

5. MODELLING TRANSPORT PROCESSES IN CHEMICAL REACTORS

In the mid 1990s, motivated through discussions with my Groningen colleague Alex Hoffmann from the Department of Chemical Engineering, I became interested in modeling of particle transport in fluidized bed reactors. This is a type of reactor that is very commonly used in the chemical engineering industry, e.g. in fluidized catalytic cracking of oil. In fluidized beds, a particulate material is made to behave fluid-like by a steady flow of gas that is entering the reactor through a diffusion plate at the bottom. Alex Hoffmann had proposed a discrete Markov model for transport in such reactors that incorporated the main physical effects governing particle transport in these reactors; see Figure 1. In Dehling, Hoffmann & Stuut (1999) we had analyzed this model and among other things proposed a continuous time model as a diffusion limit of the discrete model. Using heuristic arguments we could identify the Fokker-Planck equation of the continuous time model. However, we could not give a direct description of the continuous time process nor could we rigorously prove convergence towards this process. It was in 1997, during one of André's frequent spring break visits to Europe, that we began to discuss this problem. As so often in the past, it was one of André's great ideas that helped us solve both problems at the same time.

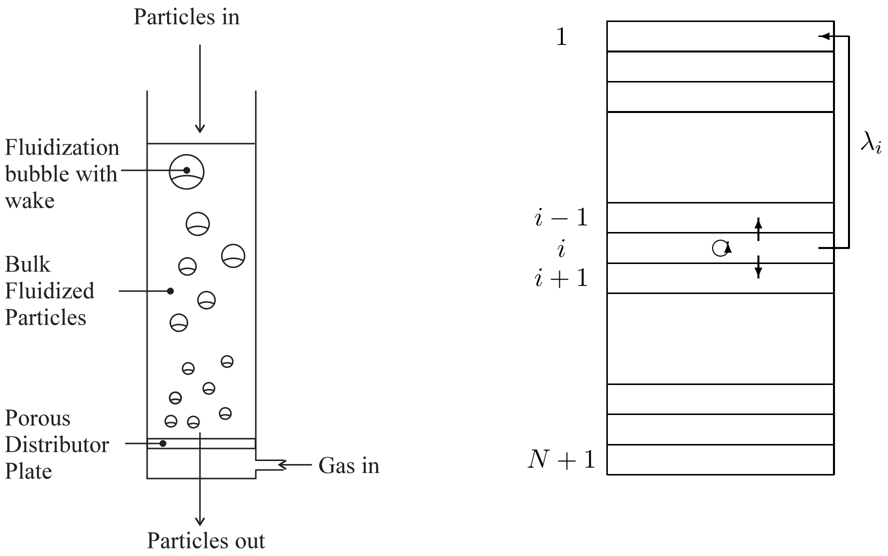

Figure 1: Physical processes governing transport of particles in fluidized bed reactors (left) and discrete Markov model for particle transport (right).

In Dehling, Hoffmann & Stuut (1999) we considered a continuously operated fluidized bed, in which particles enter at the top and are removed finally at the bottom. We studied a discrete Markov model for the distance of a single particle from the top of the reactor. In order to obtain such a model, we decomposed the reactor into N horizontal cells of equal height, with indices $1, \ldots, N$. In addition, we introduced a state $N+1$, for the exit of the reactor. The transition probabilities in the interior of the reactor are

$$\begin{aligned} p_{i,i-1} &= \delta_i \, (1-\lambda_i) \\ p_{i,i} &= \alpha_i \, (1-\lambda_i) \\ p_{i,i+1} &= \beta_i \, (1-\lambda_i) \\ p_{i,1} &= \lambda_i, \end{aligned}$$

where $\alpha_i + \beta_i + \delta_i = 1$ and $0 \leq \lambda_i \leq 1$. These transitions reflect the three physical phenomena that are commonly believed to govern particle transport in fluidized bed reactors, namely particle flow towards the bottom as a result of the continuous removal of particles, dispersion as a result of disturbance by rising fluidization bubbles and finally transport of particles to the top of the reactor in the wake of rising fluidization bubbles. Note that this Markov model is a birth-death process with additional jumps, the jump probabilities being given by λ_i. Moreover, we specified the boundary conditions

$$\begin{aligned} p_{1,1} &= 1-\beta_1(1-\lambda_1) \\ p_{1,2} &= \beta_1(1-\lambda_1) \\ p_{N+1,N+1} &= 1. \end{aligned}$$

These boundary conditions are reflecting at the top and absorbing at the bottom, in accordance with the physical mechanisms present in fluidized bed reactors.

Our continuous time model was motivated by the usual approximation of a diffusion process by a birth-death process. Given the drift $v(x)$, diffusion $D(x)$ and jump rate $\lambda(x)$, we consider for each $\Delta > 0$ the Markov process $(\tilde{X}_n^\Delta)_{n\geq 0}$ on the state space $\{1, 2, \ldots, \left[\frac{1}{\Delta}\right], \left[\frac{1}{\Delta}\right] + 1\}$ with transition probabilities

$$\begin{aligned} p_{i,i+1}^\Delta &= \left(\frac{\epsilon}{2\Delta^2} D(i\,\Delta) + \frac{\epsilon}{2\Delta} v(i\,\Delta)\right)(1 - \epsilon\,\lambda(i\,\Delta)) \\ p_{i,i-1}^\Delta &= \left(\frac{\epsilon}{2\Delta^2} D(i\,\Delta) - \frac{\epsilon}{2\Delta} v(i\,\Delta)\right)(1 - \epsilon\,\lambda(i\,\Delta)) \\ p_{i,i}^\Delta &= \left(1 - \frac{\epsilon}{\Delta^2} D(i\,\Delta)\right)(1 - \epsilon\lambda(i\,\Delta)) \\ p_{i,1}^\Delta &= \epsilon\,\lambda(i\,\Delta). \end{aligned}$$

In addition, the boundary conditions are adapted from the discrete model. We then introduce the rescaled process

$$X_t^\Delta := \Delta \cdot \tilde{X}_{[t/\epsilon]}^\Delta, \quad t \geq 0.$$

Dehling, Hoffmann & Stuut (1999) gave heuristic arguments showing that, as $\Delta \to 0$, this process converges towards a jump-diffusion process. The density of $p(t, x)$ of X_t obeys the Fokker-Planck equation

$$\frac{\partial}{\partial t} p(t, x) = \frac{1}{2} \frac{\partial^2}{\partial x^2} \left(D(x) p(t, x)\right) - \frac{\partial}{\partial x} \left(v(x) p(t, x)\right) - \lambda(x) p(t, x).$$

What we lacked, was a probabilistic description of the limit process and a rigorous proof of convergence.

André suggested to decompose the process $(\tilde{X}_n^\Delta)_{n\geq 0}$ into two parts, namely the birth-death part and the jump part, and to investigate the limit behavior of each of these parts separately. Formulating this decomposition as a program for simulation of the process $(\tilde{X}_n^\Delta)_{n\geq 0}$, we get the following three-step procedure:

1. Simulate a birth-death process $(\tilde{Y}_n^\Delta)_{n\geq 0}$ with the same transition probabilities as above, but with $\lambda(x) \equiv 0$
2. Given the process $(\tilde{Y}_n^\Delta)_{n\geq 0}$, simulate for each $n \geq 0$ coin tosses with probability $\epsilon\lambda(\Delta \cdot \tilde{Y}_n^\Delta)$ for "Head". Stop the birth-death process at the time τ^Δ of first "Head".
3. Restart the entire process afresh at time τ^Δ.

Given this decomposition, we had to study the limit behavior of the birth-death processes and the stopping times, as $\Delta \to 0$. It is well-known from the literature that the rescaled birth-death process

$$Y_t^\Delta := \Delta \cdot \tilde{Y}_{[t/\epsilon]}^\Delta, t \geq 0$$

converges in distribution towards a diffusion process $(Y_t)_{t\geq 0}$ with drift $v(x)$ and diffusion $D(x)$. Applying the Strassen-Dudley theorem, we can actually find a version of the diffusion process that is with large probability pathwise close to the birth-death process. More precisely, given $\eta > 0$ we get for Δ sufficiently small

$$P\left(\sup_{0\leq t\leq a} \left|Y_t^\Delta - Y_t\right| \geq \eta\right) \leq \eta.$$

The fact that we can couple the processes pathwise, is very important in the second step, because the law of the stopping times is determined conditionally given the path of the Y-process.

Concerning the stopping times, some straightforward calculations together with the pathwise closeness of Y and the Y^Δ-processes yields

$$P(\tau^\Delta \geq t) = \prod_{k=1}^{[t/\epsilon]} \left(1 - \epsilon\,\lambda(\Delta \cdot \tilde{Y}_k^\Delta)\right) \to \exp(-\int_0^t \lambda(Y_s) ds).$$

The limit probability is the distribution of the waiting time for the first event in a Poisson-process with intensity function $\lambda(Y_s), s \geq 0$.

Taking both parts together, we get the following recipe for simulation of a jump-diffusion process $(X_t)_{t\geq 0}$ that will eventually be the limit of the $(X_t^\Delta)_{t\geq 0}$-processes:

1. Simulate a diffusion process $(Y_t)_{t\geq 0}$ with drift $v(x)$ and diffusion $D(x)$.
2. Given the process $(Y_t)_{t\geq 0}$, simulate inhomogeneous Poisson-process with intensity $\lambda(Y_s)$ and stop the process $(Y_t)_{t\geq 0}$ at time τ of first event.
3. Start the process afresh at time τ.

That this all works, took quite some pages of calculations. In the end we were able to establish the following theorem.

Theorem 10. (Dabrowski & Dehling 1998) *With the same notation as above, for any fixed T, as $\Delta \to 0$,*

$$\left(X_t^\Delta\right)_{0\leq t\leq T} \xrightarrow{\mathcal{D}} (X_t)_{0\leq t\leq T}$$

in $D[0,T]$, equipped with the Skorohod topology.

Beginning in 1999, Chutima Dechsiri, a Groningen Ph.D. student jointly supervised by Alex Hoffmann and myself, performed some very interesting experiments where she was able to track the path of individual particles inside a fluidized bed reactor; see Dechsiri (2004). The results of these experiments confirmed parts of the model discussed above, but also showed that the model for the transport in the wakes of rising fluidization bubbles is oversimplified. First, particles are not always transported all the way to the top, but they can also be dropped along the way. Secondly, the wake transport is not happening instantaneously and the particles follow a stochastic process also during the wake transport. In an attempt to get more realistic models, Gottschalk, Dehling & Hoffmann (2008) proposed a two-phase Markov process as model for particle transport. Gottschalk (2008) showed that a generalization of the ideas developed in Dabrowski & Dehling (1998) can also be used to analyze the diffusion limit of these processes.

During André's last visit to Bochum, in the spring of 2003, we started our final joint project which until today has remained unfinished. The project was in cooperation with Chutima Dechsiri and Alex Hoffmann, who had experimental data on mixing and segregation of particles in a batch operated fluidized bed containing a binary mixture of particles. In batch operation, there is no inflow or outflow of particles during the process. In the experiments performed by Dechsiri and Hoffmann, two types of particles were in the reactor and the main interest was in the pattern of mixing or segregation of the particles during the operation. We studied an interacting particle model that takes into account the fact that no two particles can occupy the same space in the reactor. Some heuristic calculations showed that our model was able to predict the mixture density of the particles, but we never got to work out the mathematical details.

6. FAREWELL TO A GREAT FRIEND

The friendship and collaboration with André Dabrowski has been one of the most rewarding experiences of my academic career. There have occasionally been long phases when we did not meet or communicate, because both of us were busy with our own affairs. But all the time we knew that we could rely on each other. In all the years of collaboration there was never a moment of unfriendly competition. Our collaboration took place in an atmosphere of deep mutual respect. Having attended graduate school together, we knew each other's strengths and weaknesses and we felt no need to impress each other. Both of us being deeply rooted in the Christian faith, we also knew that there were more important things in life. I admired André for his bold ideas, for his adventurous spirit, for his confidence that things would eventually go well, and even more for his ability to manage time and to balance professional and family life. I have known a great man.

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